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Combined Estimators as Alternative to Feasible Generalized Least Square Estimators

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Abstract: Although the performances of the Feasible Generalized Least Square (FGLS) estimators developed to tackle violation of homoscedacity variance in linear regression model are asymptotically equivalent, their performances in small sample sizes still pose research challenges. In this paper, two FGLS estimators, CORC and ML estimators were combined with the estimator based on Principal Component (PC) Analysis and the Mean Square Error (MSE) sampling property criterion was used to examine and compare their performances through Monte Carlo Simulation study with both normally and uniformly distributed variables as regressors. The estimators were ranked at each level of autocorrelations and sample sizes and the sum of their ranks as well as the number of times each estimator has the minimum MSE was obtained. Results show that out of all the combined estimators proposed, the CORCPC123 and MLPC123 generally performed better than or compete with their separate counterpart. They are asymptotically equivalent. At small sample size (n=10), the proposed estimator CORCPC123 is conspicuously more efficient than CORC; and with uniformly distributed regressors, the CORCPC12 is best at high level of negative autocorrelation. At low level of autocorrelation, the OLS estimator is generally most efficient while the PC12 is best with uniformly distributed regressors when the sample size is small (n=10).

Keywords: OLS Estimator, FGLS Estimators, Combined Estimators, Sampling Properties, Linear Regression Model.

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1 Introduction

The Generalized Linear Model resulting from violation of the assumption of homoscedastic variance of the classical linear regression model and loss of efficiency of the OLS estimator being used to estimate its parameters led to the development of the Generalized Least Square (GLS) estimator by Aitken [1]. This GLS estimator βgiven as $\beta=(X^1\Omega^1X)^{-1}X^1\Omega^1Y$ is efficient among the class of linear unbiased estimators of β with variance – covariance matrix of β given as $V(\beta) = \sigma^2(X^1\Omega^1X)^{-1}$, where Ω is assumed to be known. The GLS estimator described requires Ω, and in particular ρ to be known before the parameters can be estimated. Thus, in linear model with autocorrelated error terms having AR (1):

$$\hat{\beta}_{(GLS)} = (X^1\Omega^{-1}X)^{-1}X^1\Omega^{-1}Y \quad (1)$$

$$V(\hat{\beta}_{(GLS)}) = \sigma^2(X^1\Omega^{-1}X)^{-1} \quad (2)$$

where

$$E(UU') = \sigma^2\Omega = \sigma^2\begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-2} & \rho^{n-1} \\ \rho & 1 & \rho & \cdots & \rho^{n-3} & \rho^{n-2} \\ .\rho^2 & \rho & 1 & \cdots & \rho^{n-4} & \rho^{n-3} \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ \rho^{n-2} & \rho^{n-3} & \rho^{n-4} & \cdots & 1 & \rho \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & \rho & 1 \end{bmatrix}$$

and $\sigma^2 = \sigma^2{}_u = \dfrac{\sigma^2{}_\varepsilon}{(1-\rho^2)}$, and the inverse of Ω is

$$\Omega^{-1} = \frac{1}{(1-\rho^2)}\begin{bmatrix} 1 & -\rho & 0 & \cdots & 0 & 0 \\ -\rho & 1+\rho^2 & -\rho & \cdots & 0 & 0 \\ 0 & -\rho & 1+\rho^2 & \cdots & 0 & 0 \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ 0 & 0 & 0 & \cdots & 1+\rho^2 & -\rho \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix}$$

Now with a suitable (n-1) x n matrix transformation P* defined by

$$P^* = \begin{bmatrix} -\rho & 1 & 0 & \cdots & 0 & 0 \\ 0 & -\rho & 1 & \cdots & 0 & 0 \\ 0 & 0 & -\rho & \cdots & 0 & 0 \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix}_{(n-1)+n}$$

Multiplying then shows that $P^{*\prime}P^*$ gives an n x n matrix which, apart from a proportional constant, is identical with Ω^{-1} except for the first elements in the leading diagonal, which is ρ^2 rather than unity. With another n x n transformation matrix P obtained from P* by adding a new row with

$\sqrt{1-\rho^2}$ in the first position and zero elsewhere, that is

$$P = \begin{bmatrix} (1-\rho^2)^{\frac{1}{2}} & 0 & 0 & \cdots & 0 & 0 \\ -\rho & 1 & 0 & \cdots & 0 & 0 \\ 0 & -\rho & 1 & \cdots & 0 & 0 \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix}_{nxn}$$

Multiplying shows that $P'P = (1-\rho^2)\Omega^{-1}$. The difference between P* and P lies only in the treatment of the first sample observation. However, when n is large, the difference is negligible, but in small sample, the difference can be major [2, 3].

The GLS estimation in (1) and (2) requires that Ω or more precisely ρ to be known but this is not often the case as ρ (or hence Ω) is always estimated via the transformation matrix P* and P and the use of the OLS estimator to get a consistent estimator $\hat{\rho}$ and have a Feasible Generalized Least Squares Estimator (FGLS).There are several ways of consistently estimating ρ using either the P* or P transformation matrix [4]. Several developed FGLS estimators include the estimator provided by Cochrane and Orcutt [5], Paris and Winstern [6], Hildreth and Lu [7], Durbin [8], Theil [9], the Beach and Mackinon [10] and Thornton [11]. Among others, the Maximum Likelihood and Maximum Likelihood Grid proposed by Beach and Mackinon [10] impose stationary by constraining the serial correlation coefficient to be between -1 and 1 and keep the first observation for estimation while that of Cochrane and Orcutt and Hildreth and Lu drop the first observation. Rao and Griliches [12] did one of the earliest Monte-Carlo investigations on the small sample properties of several two-stage regression methods in the context of autocorrelated error terms. His findings, among other things, pointed out the inefficiency of these estimators especially the CORC estimator when the sample size is small. The Principal Component Analysis suggested by Massy [13] is being used for data reduction and also as a method of estimation of model parameters in the presence of multicolliearity [14, 15, 16].

In view of the fact that the feasible generalized least square estimators, especially the CORC estimator, is inefficient in small sample size, this paper attempts to improve the efficiency of these estimators by combining them with the estimator based on Principal Component Analysis and examines the mean Square error sampling property of the resulting estimators.

2 MATERIALS AND METHODS

Consider the linear regression model of the form:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + U_t \qquad (3)$$

Where $U_t = \rho U_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma^2)$, t = 1, 2, 3,...n.

The technique adopted for the development of the combined estimator is very much similar to that of the Principal Component Estimator when used to solve multicollinearity problem. Just like the Principal Component does its estimation using the OLS estimator by regressesing the extracted components (PCs) on the standardized dependent variable, the combined estimators use the FGLS estimators, Cochrane and Orcutt (CORC) estimator [5] and the Maximum Likelihood (ML) estimator [10], by regressesing the extracted components (PCs) on the standardized dependent variable. Unlike the OLS estimator which results back into the OLS estimator when all the PCs are used [15, 16]; advantageously, since the FGLS estimators require an iterative methodology for its estimation, the proposed combined estimators may not result back into the FGLS feasible estimators when all the possible PCs are used for the estimation. Consequently, the parameters of (3) are estimated by the following eleven (11) estimators: OLS, PC1, PC12, CORC, CORCPC1, CORCPC12, CORCPC123, ML, MLPC1, MLPC12 and MLPC123 estimators.

For the Monte-Carlo simulation study, two types of regessors namely, $X_i \sim N(0,1)$ and $X_i \sim U(0,1)$ were used.The parameters of equation (3) were specified and fixed as β_0 = 4, β_1 = 2.5, β_2 = 1.8 and β_3 = 0.6. Furthermore, the experiment was replicated in 1000 times (R =1000) under four (4) levels of sample sizes (n =10, 20, 30 and 100) and twenty – one levels of autocorrelation ($\rho = -0.99, -0.9, -0.8, \ldots, 0.8, 0.9, 0.99.$). The estimators were evaluated and compared using the Mean Square Error Criterion since the separate estimator especially CORC has been reported biased in small sample size Rao and Griliches [12]. Mathematically, for any estimator $\hat{\beta}_i$ of β_i i = 0, 1, 2, 3 the Mean Square Error (MSE) property of the estimator is defined as:

$$MSE(\hat{\beta}_i) = \frac{1}{R}\sum_{j=1}^{R}\left(\hat{\beta}_{ij} - \beta_i\right)^2 \qquad (4)$$

For all these estimators, a computer program was written using Time Series Processor [17] software to evaluate Mean Square Error of the estimators on the parameters of the model. The estimators were ranked at each particular level of autocorrelations, sample sizes and the model parameters and the sum of the ranks as well as the number of times each estimator has minimum MSE was used as a basis to identify the best estimator. An estimator is best at each particular level of autocorrelation and sample size if the sum of ranks is minimum and / or has highest number of minimum MSE.

3 RESULTS AND DISCUSSION

The results of the performances of five (5) generally competing estimators are graphically represented for the normally distributed regressors. These are OLS, CORC, CORCPC123, ML and ML PC123. Figures 1A-1D and, 2A AND 2Bshow the graphical representation of the performances of the estimators with normally distributed regessorson β_1, β_2 and β_3 respectively. The performances

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of the estimators with uniform regressors follow the same pattern even though the estimators perform much better with normal regressors (see Figure 1 and 2). From the figures, the OLS estimator is best at low level of autocorrelation while the other ones only compete especially with increased sample size. This has also been noted and reported by many authors including Rao and Griliches [12] and Ayinde and Olaomi [18]. Furthermore, it can be easily seen that the estimators differ very significantly at small sample size, n=10, and that of CORC and ML with their proposed counterparts perform equivalently as the sample size increases. When the sample size is small (n=10), the alternative combined estimator CORCPC123 is more efficient than the CORC while the MLPC123 is slightly more efficient especially when the autocorrelation level is positive. The performances of the estimators are also affected by different type of regressors.

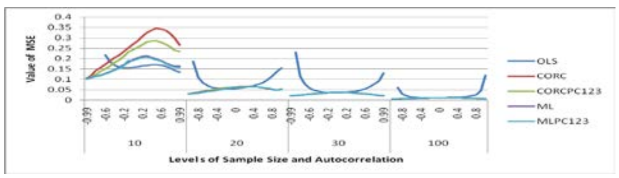

Fig. 1: Graphical Representation of the Mean Square Error of β_1 of some of the estimators with normally distributed regressors.

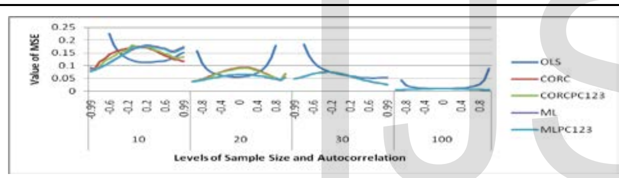

Fig. 2: Graphical Representation of the Mean Square Error of β_2 of some of the estimators with normally distributed regressors.

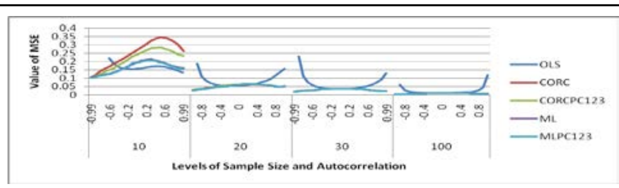

Fig. 3: Graphical Representation of the Mean Square Error of β_3 of some of th estimators with normally distributed regressors.

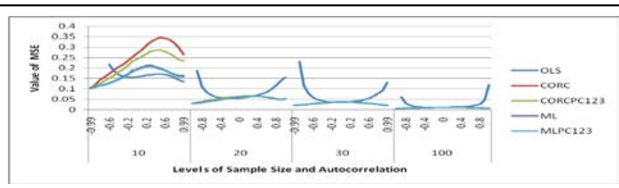

Fig. 4: Graphical Representation of the Mean Square Error of β_1 of some of th estimators with uniformlly distributed regressors.

The results of the performances of eight (8) fair ones out of eleven (11) considered, having ranked their Mean Square Error at each parameter level, summing their ranks over the model parameters and obtaining the number of times each estimator has the minimum MSE, at various levels of autocorrelation and sample size are presented in Table 2A and 2B. and 3.These estimators are OLS, PC1, CORC, ML, CORCPC12, CORCPC123, MLPC12 and MLPC123.

Table 1A: Summary Table of the total ranks of the estimators at various levels of autocorrelation and sample size.

Normal Regressors												
Sample Size	Estimators	Autocorrelation value										
		-0.99	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0
10	OLS	23	29	24	23	21	17	14	8	**4**	**4**	**4**
	PC12	28	24	28	29	29	26	24	22	21	18	18
	CORC	18	20	20	20	20	22	24	26	26	24	26
	CORCPC12	22	13	24	25	26	28	29	27	27	26	26
	CORCPC123	**8**	11	11	11	11	12	13	18	19	21	20
	ML	13	**4**	7	7	9	10	12	15	16	14	17
	MLPC12	23	24	24	23	24	25	24	22	21	22	22
	MLPC123	**8**	9	**6**	**6**	**4**	**4**	**4**	**6**	10	15	11
20	OLS	24	20	20	20	20	18	15	8	**4**	**4**	**4**
	PC12	30	28	28	28	28	28	28	25	25	27	28
	CORC	14	14	16	16	16	18	18	20	20	20	20
	CORCPC12	26	29	29	29	30	30	30	31	29	27	27
	CORCPC123	11	11	11	11	11	12	14	16	16	16	16
	ML	**7**	8	7	7	9	8	9	10	12	12	12
	MLPC12	24	27	27	27	26	26	26	28	30	30	29
	MLPC123	8	**7**	**6**	**6**	**4**	**4**	**4**	**6**	8	8	8
30	OLS	23	23	23	23	23	21	20	20	15	**4**	**4**
	PC12	27	27	27	27	27	25	25	25	25	22	22
	CORC	10	8	11	11	13	15	17	17	18	21	21
	CORCPC12	27	27	28	28	28	30	29	29	29	29	29
	CORCPC123	11	10	11	10	13	13	11	12	14	17	17
	ML	11	13	**7**	**8**	**6**	**6**	**6**	**6**	8	11	12
	MLPC12	27	27	26	26	26	28	29	29	29	29	29
	MLPC1233	**8**	**9**	11	11	8	**6**	7	**6**	**6**	11	10
100	OLS	21	23	23	20	20	18	18	18	18	10	**6**
	PC12	29	27	27	27	26	26	27	26	25	26	27
	CORC	**9**	13	13	10	11	13	13	13	12	14	15
	CORCPC12	25	27	27	28	29	29	29	29	29	29	29
	CORCPC123	**9**	12	12	**9**	10	13	13	13	14	16	19
	ML	12	8	8	12	**9**	**7**	**8**	**8**	**8**	**8**	7
	MLPC12	27	27	27	29	29	29	28	29	29	29	28
	MLPC123	12	**7**	**7**	**9**	10	9	**8**	**8**	9	12	13

Table 1B: Summary Table of the total ranks of the estimators at various levels of autocorrelation and sample size

Sample Size	Estimators	Normal Regressors										
		Autocorrelation value										
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
10	OLS	**4**	**4**	**4**	**4**	**5**	**6**	**8**	10	12	12	11
	PC12	18	18	18	18	19	20	20	21	21	21	20
	CORC	26	23	23	23	23	23	23	23	22	22	19
	CORCPC12	26	26	26	25	26	28	28	28	28	28	29
	CORCPC123	20	20	19	20	19	19	19	19	18	18	13
	ML	17	19	19	19	19	17	16	11	11	11	16
	MLPC12	22	22	22	22	20	21	22	23	22	22	27
	MLPC123	11	12	13	13	13	10	**8**	**9**	**10**	**10**	**9**
20	OLS	**4**	**4**	**4**	8	14	16	16	16	17	17	17
	PC12	28	28	28	28	28	27	27	27	27	24	23
	CORC	20	20	20	19	16	18	20	17	16	16	15
	CORCPC12	27	27	27	27	27	26	26	27	27	28	28
	CORCPC123	16	15	15	15	12	11	13	12	13	14	13
	ML	12	13	13	11	11	11	**6**	**8**	**6**	**6**	**10**
	MLPC12	29	29	29	29	29	28	28	27	27	28	26
	MLPC123	8	8	8	**7**	**7**	**7**	8	10	11	11	12
30	OLS	**4**	**5**	14	19	19	19	17	17	16	18	20
	PC12	22	22	26	25	25	25	26	25	25	28	27
	CORC	21	21	18	17	17	17	15	13	14	13	16
	CORCPC12	29	29	29	29	29	29	29	28	28	24	25
	CORCPC123	17	17	16	12	11	11	12	11	13	13	16
	ML	12	12	8	**6**	9	9	9	16	10	15	**7**
	MLPC12	29	29	29	29	29	29	29	28	28	24	22
	MLPC1233	10	9	**4**	7	**5**	**5**	**7**	**6**	**10**	**9**	11
100	OLS	**6**	10	19	20	20	20	20	20	20	18	20
	PC12	27	29	30	30	30	29	29	29	28	28	24
	CORC	15	18	16	16	15	13	13	13	11	16	12
	CORCPC12	29	28	27	27	27	26	26	26	29	25	26
	CORCPC123	19	13	11	11	11	11	9	9	**7**	13	12
	ML	7	**9**	**7**	8	9	16	15	18	18	12	13
	MLPC12	28	27	27	25	25	24	24	24	23	23	26
	MLPC123	13	10	**7**	**7**	**7**	**5**	**8**	**5**	10	**9**	**11**

Table 1C: Summary Table of the total ranks of the estimators at various levels of autocorrelation and sample size.

Sample Size	Estimators	Uniform Regressors										
		Autocorrelation value										
		-0.99	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0
10	OLS	26	31	31	30	27	27	24	15	12	11	11
	PC12	27	28	29	26	22	20	10	**6**	**4**	**4**	**4**
	CORC	17	21	22	27	21	30	32	32	32	32	32
	CORCPC12	23	20	22	21	20	21	20	20	17	17	17
	CORCPC123	**8**	13	13	15	15	20	25	27	27	27	26
	ML	10	**4**	9	7	16	13	16	21	24	20	24
	MLPC12	21	19	13	13	12	**4**	**5**	7	9	10	10
	MLPC123	12	8	**5**	**5**	**11**	9	12	16	19	23	20
20	OLS	29	29	24	24	24	24	18	10	**4**	**4**	**4**
	PC12	31	29	30	30	30	30	29	26	22	18	18
	CORC	15	16	16	16	17	18	20	23	23	24	24
	CORCPC12	23	23	26	27	27	27	30	30	32	32	32
	CORCPC123	10	8	10	12	12	12	14	19	19	19	19
	ML	**4**	11	9	8	8	7	8	9	13	13	13
	MLPC12	21	23	24	23	22	21	21	22	22	25	25
	MLPC123	11	**5**	**5**	**4**	**4**	**5**	**4**	**5**	9	9	9
30	OLS	27	23	23	23	23	23	23	19	15	**4**	**4**
	PC12	27	27	27	27	27	27	27	27	27	22	22
	CORC	11	13	13	**7**	**7**	12	14	17	18	21	21
	CORCPC12	24	26	26	26	26	26	27	27	27	29	29
	CORCPC123	11	**7**	11	10	8	14	12	13	14	17	17
	ML	**7**	11	**7**	10	14	**4**	8	**7**	**8**	12	12
	MLPC12	26	28	28	28	28	28	27	27	27	29	29
	MLPC1233	11	9	9	13	11	10	**6**	**7**	**8**	10	10
100	OLS	30	32	32	32	32	31	28	24	22	**10**	**5**
	PC12	28	28	28	28	20	29	30	32	30	27	19
	CORC	16	16	16	16	16	16	15	18	17	20	22
	CORCPC12	23	23	23	22	22	22	23	22	22	24	26
	CORCPC123	12	12	12	12	12	12	13	14	14	20	20
	ML	7	7	7	**6**	7	7	**5**	**6**	**6**	**10**	11
	MLPC12	21	21	21	22	22	22	23	22	22	23	26
	MLPC123	**5**	**5**	**5**	**6**	**5**	**5**	7	**6**	10	**10**	15

Table 1D: Summary Table of the total ranks of the estimators at various levels of autocorrelation and sample size

Sample Size	Estimators	Uniform Regressors										
		Autocorrelation value										
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
10	OLS	11	11	12	13	14	14	16	16	20	20	20
	PC12	**4**	**4**	**4**	**4**	**5**	**5**	**8**	10	12	14	16
	CORC	32	32	32	29	29	29	29	27	26	26	25
	CORCPC12	17	17	15	15	13	12	11	11	**10**	**9**	**9**
	CORCPC123	26	25	24	25	25	25	25	25	21	22	22
	ML	24	25	23	26	26	26	25	24	23	22	21
	MLPC12	10	10	11	10	10	11	9	**9**	11	11	12
	MLPC123	20	20	22	22	22	22	21	22	21	20	19
20	OLS	**4**	**4**	**5**	13	17	23	25	25	25	25	25
	PC12	18	19	19	23	25	26	29	27	27	27	25
	CORC	24	23	23	22	20	17	18	20	20	17	15
	CORCPC12	32	32	31	31	31	30	26	26	26	25	26
	CORCPC123	19	19	19	17	13	12	12	12	12	12	9
	ML	13	12	12	7	7	**6**	**4**	**4**	**4**	**5**	**7**
	MLPC12	25	25	25	26	26	24	22	22	22	23	24
	MLPC123	9	10	10	**5**	**5**	**6**	8	8	8	10	13
30	OLS	**4**	**4**	14	20	22	22	22	21	21	21	21
	PC12	22	22	23	26	28	28	28	29	29	29	29
	CORC	21	21	19	16	15	15	15	17	12	16	13
	CORCPC12	29	29	30	29	27	27	27	25	24	24	25
	CORCPC123	17	17	15	11	11	11	11	12	**7**	13	**9**
	ML	12	12	9	8	8	8	6	**5**	10	**6**	**9**
	MLPC12	29	29	29	29	26	26	27	25	26	26	25
	MLPC1233	10	10	**5**	**5**	**7**	**7**	**8**	10	15	9	13
100	OLS	**5**	18	25	27	29	30	30	30	31	30	26
	PC12	19	25	29	31	31	30	30	30	29	30	24
	CORC	22	21	19	16	15	15	12	11	9	13	13

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	CORCPC12	26	22	20	23	22	22	22	20	20	20	23
	CORCPC123	20	16	15	11	11	11	10	**8**	**6**	9	14
	ML	11	**10**	**6**	7	**6**	**7**	12	13	13	16	11
	MLPC12	26	22	22	23	23	22	22	22	23	20	23
	MLPC123	15	**10**	8	**6**	8	**7**	**6**	10	14	**6**	**10**

Table 2A: Number of time at which the estimators have minimum Mean Square Error

Sample Size	Autocorrelation value	Normally Distributed Regressors						
		Estimators						
		OLS	CORC	CORCPC123	ML	MLPC123	OTHERS	BEST
10	-0.99	0	0	1	1	2	0	MLPC123
	-0.9	0	0	0	4	0	0	ML
	-0.8	0	0	0	1	3	0	MLPC123
	-0.7	0	0	0	1	3	0	MLPC123
	-0.6	0	0	0	0	4	0	MLPC123
	-0.5	0	0	0	0	4	0	MLPC123
	-0.4	0	0	0	0	4	0	MLPC123
	-0.3	2	0	0	0	2	0	OLS MLPC123
	-0.2	4	0	0	0	0	0	OLS
	-0.1	4	0	0	0	0	0	OLS
	0	4	0	0	0	0	0	OLS
	0.1	4	0	0	0	0	0	OLS
	0.2	4	0	0	0	0	0	OLS
	0.3	4	0	0	0	0	0	OLS
	0.4	3	0	0	0	1	0	OLS
	0.5	3	0	0	1	0	0	OLS
	0.6	2	0	0	2	0	0	OLS/ML
	0.7	2	0	1	1	0	0	OLS
	0.8	1	1	1	1	0	0	ML
	0.9	1	1	1	1	0	0	ML
	0.99	1	1	0	2	0	0	ML
20	-0.99	0	0	0	2	2	0	ML/ MLPC123
	-0.9	0	0	0	1	3	0	MLPC123
	-0.8	0	0	0	1	3	0	MLPC123
	-0.7	0	0	0	1	3	0	MLPC123
	-0.6	0	0	0	0	4	0	MLPC123
	-0.5	0	0	0	0	4	0	MLPC123
	-0.4	0	0	0	0	4	0	MLPC123
	-0.3	2	0	0	0	2	0	OLS/ MLPC123
	-0.2	4	0	0	0	0	0	OLS
	-0.1	4	0	0	0	0	0	OLS
	0	4	0	0	0	0	0	OLS
	0.1	4	0	0	0	0	0	OLS
	0.2	4	0	0	0	0	0	OLS
	0.3	3	0	0	0	1	0	OLS
	0.4	1	0	1	0	2	0	MLPC123
	0.5	1	0	1	0	2	0	MLPC123
	0.6	1	0	0	2	1	0	ML
	0.7	1	1	0	2	0	0	ML
	0.8	0	1	0	3	0	0	ML
	0.9	0	1	0	3	0	0	ML
	0.99	0	1	1	1	1	0	ML
30	-0.99	0	1	1	1	1	0	MLPC123
	-0.9	0	1	0	1	2	0	MLPC123
	-0.8	0	1	0	1	2	0	MLPC123
	-0.7	0	1	0	2	1	0	ML
	-0.6	0	0	0	2	2	0	ML/ MLPC123
	-0.5	0	0	0	2	2	0	ML/ MLPC123
	-0.4	0	0	0	2	2	0	ML/ MLPC123
	-0.3	0	0	0	2	2	0	ML/ MLPC123
	-0.2	1	0	0	1	2	0	MLPC123
	-0.1	4	0	0	0	0	0	OLS
	0	4	0	0	0	0	0	OLS
	0.1	3	0	0	0	1	0	OLS
	0.2	0	0	0	0	4	0	MLPC123
	0.3	0	0	0	3	1	0	ML
	0.4	0	0	0	1	3	0	MLPC123
	0.5	0	0	0	1	3	0	MLPC123
	0.6	0	0	0	2	2	0	ML/ MLPC123
	0.7	0	1	0	0	3	0	MLPC123
	0.8	1	1	0	2	0	0	ML
	0.9	1	1	0	1	1	0	ML
	0.99	0	0	1	3	0	0	ML
100	-0.99	0	1	2	0	1	0	CORCPC123
	-0.9	0	1	0	0	3	0	MLPC123
	-0.8	0	1	0	0	3	0	MLPC123
	-0.7	0	1	1	0	2	0	MLPC123
	-0.6	0	1	1	0	2	0	MLPC123
	-0.5	0	1	0	1	2	0	MLPC123
	-0.4	0	1	0	0	3	0	MLPC123
	-0.3	0	1	0	0	3	0	MLPC123
	-0.2	0	1	0	1	2	0	MLPC123
	-0.1	2	1	0	1	0	0	OLS
	0	3	1	0	0	0	0	OLS
	0.1	2	0	1	1	0	0	OLS
	0.2	0	0	1	2	1	0	ML
	0.3	0	0	1	2	1	0	ML
	0.4	0	0	1	2	1	0	ML
	0.5	0	0	1	0	3	0	MLPC123
	0.6	0	1	1	1	1	0	MLPC123
	0.7	0	0	0	1	3	0	MLPC123
	0.8	0	1	0	1	2	0	MLPC123
	0.9	1	0	0	2	1	0	ML
	0.99	0	2	0	1	1	0	CORC

Table 2B: Number of time at which the estimators have

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minimum Mean Square Error

Sample Size	Autocorrelation value	OLS	PC12	CORC	CORCPC12	CORCPC123	ML	MLPC12	MLPC123	BEST
		Uniformly Distributed Regressors								
		Estimators								
10	-0.99	0	0	0	0	1	3	0	0	ML
	-0.9	0	0	0	0	0	4	0	0	ML
	-0.8	0	0	0	0	0	0	1	3	MLPC123
	-0.7	0	0	0	0	0	1	0	3	MLPC123
	-0.6	0	0	0	0	1	0	2	1	MLPC12
	-0.5	0	0	0	0	0	0	4	0	MLPC12
	-0.4	0	1	0	0	0	0	3	0	MLPC12
	-0.3	0	3	0	0	0	0	1	0	PC12
	-0.2	0	4	0	0	0	0	0	0	PC12
	-0.1	0	4	0	0	0	0	0	0	PC12
	0	0	4	0	0	0	0	0	0	PC12
	0.1	0	4	0	0	0	0	0	0	PC12
	0.2	0	4	0	0	0	0	0	0	PC12
	0.3	0	4	0	0	0	0	0	0	PC12
	0.4	0	3	0	1	0	0	0	0	PC12
	0.5	0	3	0	1	0	0	0	0	PC12
	0.6	0	1	0	2	0	0	1	0	CORCPC 12
	0.7	0	1	0	2	0	0	1	0	CORCPC 12
	0.8	0	1	0	2	0	0	1	0	CORCPC 12
	0.9	0	1	0	2	0	0	1	0	CORCPC 12
	0.99	0	1	1	1	0	0	1	0	CORCPC 12
20	-0.99	0	0	0	0	0	4	0	0	ML
	-0.9	0	0	0	0	1	0	0	3	MLPC123
	-0.8	0	0	0	0	1	0	0	3	MLPC123
	-0.7	0	0	0	0	0	0	0	4	MLPC123
	-0.6	0	0	0	0	0	0	0	4	MLPC123
	-0.5	0	0	0	0	0	1	0	3	MLPC123
	-0.4	0	0	0	0	0	0	0	4	MLPC123
	-0.3	1	0	0	0	0	0	0	3	MLPC123
	-0.2	4	0	0	0	0	0	0	0	OLS
	-0.1	4	0	0	0	0	0	0	0	OLS
	0	4	0	0	0	0	0	0	0	OLS
	0.1	4	0	0	0	0	0	0	0	OLS
	0.2	3	1	0	0	0	0	0	0	OLS
	0.3	0	0	0	0	0	1	0	3	MLPC123
	0.4	0	0	0	0	0	1	0	3	MLPC123
	0.5	0	0	0	0	0	2	0	2	ML/ MLPC123
	0.6	0	0	0	0	0	4	0	0	ML
	0.7	0	0	0	0	0	4	0	0	ML
	0.8	0	0	0	0	0	4	0	0	ML
	0.9	0	0	0	0	0	3	0	1	ML
	0.99	0	0	1	0	1	2	0	0	ML
30	-0.99	0	0	1	0	0	2	0	1	ML
	-0.9	0	0	1	0	2	1	0	0	CORCPC 123
	-0.8	0	0	1	0	0	2	0	1	ML
	-0.7	0	0	2	0	1	1	0	0	CORC
	-0.6	0	0	1	0	2	0	0	1	CORCPC 123
	-0.5	0	0	0	0	0	4	0	0	ML
	-0.4	0	0	0	0	0	2	0	2	ML/ MLPC123
	-0.3	0	1	0	0	0	1	0	2	MLPC123
	-0.2	0	4	0	0	0	0	0	0	OLS
	-0.1	0	4	0	0	0	0	0	0	OLS
	0	0	4	0	0	0	0	0	0	OLS
	0.1	0	4	0	0	0	0	0	0	OLS
	0.2	0	1	0	0	0	0	0	3	MLPC123
	0.3	0	0	0	0	0	1	0	3	MLPC123
	0.4	0	0	0	0	1	1	0	2	MLPC123
	0.5	0	0	0	0	1	2	0	1	ML
	0.6	0	0	0	0	1	2	0	1	ML
	0.7	0	0	0	0	0	3	0	1	ML
	0.8	0	2	0	0	1	1	0	0	PC12
	0.9	0	0	0	0	0	3	0	1	ML
	0.99	0	2	0	0	1	1	0	0	PC12
10 0	-0.99	0	0	0	0	1	3	0	0	ML
	-0.9	0	0	0	0	0	1	0	3	MLPC123
	-0.8	0	0	0	0	0	1	0	3	MLPC123
	-0.7	0	0	0	0	0	2	0	2	ML/ MLPC123
	-0.6	0	0	0	0	0	1	0	3	MLPC123
	-0.5	0	0	0	0	0	1	0	3	MLPC123
	-0.4	0	0	0	0	0	3	0	1	ML
	-0.3	0	0	0	0	0	2	0	2	ML/ MLPC123
	-0.2	0	0	0	1	0	3	0	0	ML
	-0.1	2	0	0	1	0	1	0	0	OLS
	0	3	1	0	0	0	0	0	0	OLS
	0.1	0	1	0	0	0	2	0	1	ML
	0.2	0	0	0	1	0	2	0	1	ML
	0.3	0	0	0	0	0	2	0	2	ML
	0.4	0	0	0	0	1	3	0	0	ML
	0.5	0	0	0	0	0	2	0	2	ML/ MLPC123
	0.6	0	0	0	0	1	0	0	3	MLPC123
	0.7	0	0	1	0	1	1	0	1	MLPC123
	0.8	0	0	1	0	2	1	0	0	CORCPC 123
	0.9	0	0	1	0	0	0	0	3	MLPC123
	0.99	0	0	1	0	0	1	0	2	MLPC123

From Table 1A-1D and 2A-AB, it can be seen that the performances of the estimators differ with different specification of regressors especially at small sample size. For instance when n = 10 at low level of autocorrelation, the OLS is best with normally distributed regressors while PC12 is

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best with uniformly distributed regressors; and at other levels of autocorrelation, the ML or MLPC123 is best with normally distributed regressors while the MLPC12 is best with positive autocorrelation and CORCPC12 best with negative autocorrelation. These types of results have been reported many authors [19, 20, 21, 22]. Generally speaking from Table 1, the results of the CORCPC123 and MLPC123 combined estimators are better than that of their separate counterparts, CORC and ML. However, the ML estimator occasionally performs slightly better than the MLPC123 combined estimator. Furthermore from Table 2, the CORC or CORCPC123 performs best with normal regressors when the sample size is large and autocorrelation is very close to unity.Summarily, it can be said that MLPC123 is generally better to estimate parameters of autocorrelated error model.

4 Conclusion

This study has combined two Feasible Generalized Estimators with the Estimator based on Principal component Analysis and compared theirperformances their separate counterparts. The combined estimators generally perform better or compete favourably with their counterparts. At low levels of autocorrelation, the OLS estimator is often best but the PC12 estimator is equally better at low sample size. The combined estimators utilizing all the CPs components are asymptotically equivalent with the separate counterparts.Thus, the combined estimators especially that of ML and Principal Component has the advantage of producing a more efficient result when used.

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